SEC SYLLABUS (2006-7)

MATHEMATICS SEC 23

SYLLABUS

Mathematics SEC 23	Available in September (Paper 1 and Paper IIB only)
Syllabus	Paper 1: Section A (20 mins) Section B (1 hr 40 mins)+Paper II 2(hrs)

Introduction

Mathematics furnishes the prime means by which information can be organised, communicated and manipulated. It is also an ever-expanding body of facts, skills, concepts and strategies used in the solution of a wide range of problems. As a consequence, when implementing this syllabus, teachers of Mathematics should emphasize that:

(i) Mathematics is useful. It equips children with the necessary knowledge to help them understand and interact with the world around them. Moreover, it forms the basis of science, technology, architecture, engineering, commerce, industry and banking. It is also increasingly being used in the medical sciences, biological sciences, economics and geography. This pervasiveness makes Mathematics one of the most important subjects in the school curriculum.
(**Utilitarian Aspect of Mathematics Teaching and Learning**)

(ii) Mathematics is an evolving body of knowledge that is characterised by its order, precision, conciseness and logic. It should offer the children intellectual challenge, excitement, satisfaction and wonder.
(**Aesthetic Aspect of Mathematics Teaching and Learning**)

Aims

When implementing this syllabus, teachers should aim to enable candidates to

- understand and appreciate the place and purpose of Mathematics in society and apply mathematical concepts to situations arising in their own lives;
- apply mathematical knowledge and understanding to solve problems;
- think and communicate mathematically - precisely, logically and creatively;
- develop a positive attitude to Mathematics, including confidence and perseverance;
- develop an ability to work independently and co-operatively when doing Mathematics;
- appreciate the interdependence of the different branches of Mathematics;
- acquire a secure foundation for the further study of Mathematics;
- use Mathematics across the curriculum;
- make efficient, creative and effective use of appropriate technology in Mathematics.

Assessment Objectives

The examination will, in general, test

- the candidate's ability to recall, understand and apply mathematical knowledge in a wide context;
- the candidate's ability to understand and analyse a problem, select an appropriate strategy, apply suitable knowledge and techniques to solve it, verify and interpret the results;
- the candidate's ability to understand, interpret and evaluate mathematical ideas that are presented in oral, written and visual forms.

In particular, the candidate will be required to demonstrate the ability to

- communicate, conjecture, reason and prove mathematically;
- understand the nature of numbers and make use of them;
- understand the nature of algebraic relationships and make use of them;
- understand the nature and properties of shape, space and measures and make use of them;
- understand the nature of statistics and process, represent and interpret data;
- understand the nature of probability and calculate the probabilities of events.

During the course candidates should be given opportunities to

- use calculators and computer software including spreadsheets, LOGO, a dynamic geometry package and computer algebra system;
- use computers as a source of large samples, as a tool for exploring graphical representations, and as a means for simulating events;
- develop a feel for numbers
- develop and use a range of methods of computation, namely, mental, pencil-and-paper, calculator and computer methods, and apply these to a range of problems;
- develop and use a range of methods for approximation of numbers and apply these to a range of problems;
- develop and use a range of methods for estimation of measures and apply these to a range of problems;
- explore a variety of situations which lead to the expression of relationships;
- consider how relationships between number operations underpin the techniques for manipulating algebraic expressions;
- consider how algebra can be used to model real-life situations and to solve problems;
- explore shape and space through drawing and practical work;
- use computers to generate and transform graphic images and to solve problems;
- formulate questions that can be solved using statistical methods;
- undertake purposeful inquiries based on data analysis;
- engage in practical and experimental work in order to appreciate principles which govern random events;
- look critically at some of the ways in which representations of data can be misleading and conclusions can be uncertain.

Scheme of Assessment

The examination will consist of two papers, **Paper I** and **Paper II**, each of 2 hours duration. Questions will be set in English and must be answered in English.

Paper I

This paper is to be taken by all candidates and will cover the Core Syllabus content only.
It will be divided into **two Sections**, **A** and **B**.
The questions in Section A will be answered on the question paper itself.
The questions in Section B will be answered on the answer booklet provided.

Section A

- It will consist of twenty short questions to be answered in 20 minutes.
- Each question will carry 1 mark.
- **No calculators, rulers, protractors or any other mathematical instruments will be allowed**.
- Questions will typically involve numerical calculations, approximations, estimations, data and graphical interpretations, application of formulae, recall and applications of properties of shapes, recall and applications of mathematical facts
- To answer these questions, particularly those involving numerical calculations, candidates are advised to choose and use the more efficient techniques (mental and pencil-and-paper). They are expected to have a range of strategies to aid mental calculations of unknown facts from facts that can be rapidly recalled.

Section B

- It will consist of nine to eleven compulsory graded questions to be answered in one hour and forty minutes.
- The questions may have different mark allocations which will be stated on the paper and will carry a total of 80 marks.

- **Candidates are expected to use mathematical instruments and scientific calculators with statistical functions. Programmable calculators are not allowed.**

Paper II

There will be two versions of this paper (A or B). Candidates will be required to indicate on the registration form which version they wish to sit for. No change in the choice of paper will be allowed after the registration period. In the September supplementary session only Paper I and Paper IIB will be offered.

Candidates are expected to use mathematical instruments and scientific calculators with statistical functions. Programmable calculators are not allowed.

Paper IIA will consist of about nine to eleven compulsory questions with varying mark allocations per question which will be stated on the paper, carrying a total of 100 marks. The questions in this paper will cover the syllabus content for both the Core and the Extension. A typical problem in this paper will be more difficult to solve than a typical Paper I problem. The time allowed for this Paper is two hours.

Candidates who intend to further their study in Mathematics at Intermediate Level or Advanced Level are advised to sit for Paper IIA.

Paper IIB will consist of twenty to twenty-eight questions with varying mark allocations stated on the paper and will carry a total of 100 marks. A typical problem in this paper will be easier to solve than a typical Paper 1 problem. The time allowed for this Paper is two hours.

The overall weighting (±5%) for each of the four main components of the syllabus is shown below:

	Number	**Algebra**	**Shape, Space and Measures**	**Data Handling**
Core Paper and Paper IIA	30%	35%	25%	10%
Core Paper and Paper IIB	40%	20%	25%	15%

Results

Candidates sitting for **Paper I** and **Paper IIA** may qualify for Grades **1**, **2**, **3**, **4** or **5**. The results for candidates who do not obtain at least a Grade **5** shall remain Unclassified (**U**).
Candidates sitting for **Paper I** and **Paper IIB** may qualify for Grades **4**, **5**, **6**, or **7**. The results for candidates who do not obtain at least a Grade 7 shall remain Unclassified (**U**).

Grade Descriptions

The following descriptions are meant to provide a general indication of the standards of achievement normally shown by candidates earning particular grades. However the final grade awarded will reflect the extent to which the candidates have met the assessment objectives overall.

Grade 1 is awarded to candidates whose answers exhibit
- an understanding of complex non-routine problems
- logical reasoning and valid conclusions
- an overall high performance in all areas of the syllabus
- a high level of presentation (providing evidence of effective and clear communication through writing and diagrams)

- correct computations and solutions.

Grade 5 is awarded to candidates whose answers exhibit
- understanding of routine problems
- an acceptable amount of reasoning and valid conclusions
- an average performance in most areas of the syllabus
- an adequate level of presentation and communication.

Grade 7 is awarded to candidates whose answers show
- understanding of simple routine problems
- a poor performance in all areas of the syllabus
- some attempt at communication.

Table of Formulae

A table of the formulae reproduced below will be provided for the candidate's use. These formulae will be required for Paper IIA only.

Area of a Triangle	$\frac{1}{2}ab\sin C$
Curved Surface Area of Right Circular Cone	πrl
Surface Area of a Sphere	$4\pi r^2$
Volume of a Pyramid / Right Circular Cone	$\frac{1}{3}\text{base area} \times \text{perpendicular height}$
Volume of Sphere	$\frac{4}{3}\pi r^3$
Solutions of $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Syllabus

The syllabus is divided into four main areas: **Number** – **Algebra** – **Shape, Space and Measures – Data Handling**.

Candidates taking **Paper IIB** need only cover the **Core** content (shown in normal type).

Candidates taking **Paper IIA** have to cover the **Core** content (shown in normal type) and the **Extension** content (shown in **bold** type).

Examples are shown in italic.

CORE	EXTENSION
NUMBER	
1. Recognise, understand and use integers (positive or negative), factors (divisors) and multiples, least common multiple, prime numbers and prime factor decomposition.	**Highest common factor.**
2. Generate number sequences.	
3. Understand that the reciprocal of a number is its multiplicative inverse; use index notation (*e.g.* 7^3, 7^{-2}); understand the terms square, square root, cube and cube root; understand and use the index laws for multiplication and division of integer powers; understand and use the standard index form, expressed in conventional notation and on a calculator display (*e.g. carry relatively simple calculations within scientific contexts*).	**Use the index laws for positive and negative fractional powers.**
4. Understand and use fractions in real life contexts; recognise equivalent fractions; simplify fractions; order fractions; convert fractions to decimals and vice-versa; know that many simple fractions can be represented as recurring decimals; know that fractions with denominators that have only prime factors of 2 and 5 will terminate.	
5. Understand and use decimals in real life contexts; recognise terminating and recurring decimals; order decimals by using place value and by their positions on the number line.	
6. Understand and use percentages in real life contexts; interpret percentage as "number of parts per hundred", convert simple fractions of a whole to percentages of the whole and vice versa (*e.g. interpret* 10% *of* 40 *as* $10/100 \times 40$); express a quantity as a percentage of another; calculate percentage increase and decrease (*e.g. a* 15% *increase in value of C is calculated as* $1.15 \times C$ *and a* 20% *discount on* Lm250 *gives a total calculated as* $0.8 \times$ Lm250); carry out calculations involving reverse percentages (*e.g. to find the cost price given the selling price and percentage profit.*)	
7. Understand and use negative numbers in real life contexts (*e.g. finding the temperature difference between temperatures below zero*).	
8. Use ratio notation (*e.g. in maps and scale drawing*); recognise its various connections with fraction notation; reduce a ratio to its simplest form;	

divide a quantity in a given ratio. 9. Use the four rules for calculations with integers, decimals and fractions, including the correct order of operations and use of brackets. 10. Understand and use measures; use metric units of mass, length, area, volume and capacity in practical situations; express quantities in terms of larger and smaller units; calculate time in terms of the 12-hour and 24-hour clock; read and interpret clocks, dials and time-tables; read and use scales in practical situations (*e.g. read a thermometer scale*).	
11. Use money; convert from one currency to another (*e.g. change from Maltese Liri to Euros and vice-versa*); solve problems on personal and household finance, involving earnings, simple interest, tax and insurance.	**Make repeated use of a multiplier raised to a power (the growth or decay factor) to compute Compound Interest, appreciation or depreciation; use the calculator and the spreadsheet to investigate the factors affecting these; determine, by trial and error, the number of years by means of a calculator.**
12. Understand and use the elementary ideas and notation of direct and inverse proportion, including calculating an unknown quantity from quantities that vary in direct- or inverse- proportion. Understand and use the elementary ideas of common measures of rate (*e.g. calculate average speed*).	
13. Carry out estimations and approximations including making estimates of measures, rounding to a specified number of significant figures and decimal places to reasonable accuracy in the context of a given problem, making sensible approximations in calculations involving multiplication and/or division.	**Understand and use limits of accuracy; give appropriate upper and lower bounds for data given to a specified accuracy (*e.g. measured lengths*); obtain appropriate upper and lower bounds to solutions of simple problems (*e.g. the calculation of the perimeter or area of a rectangle given data to a specified accuracy*).**
14. Use calculators efficiently and effectively; know how to enter complex calculations; understand the calculator display interpreting it appropriately (*e.g. in money calculations*); know when not to round during the intermediate steps of a calculation; know how to interpret numbers displayed in standard form and know how to enter numbers in the standard form; apply appropriate checks of accuracy (*e.g. working the problem backwards from its solution or make approximations to check the*	

reasonableness of the result).	
ALGEBRA 15. Use and understand algebraic representation; use letters to express generalised numbers; understand that algebraic entities are transformed according to the well-defined properties of generalised arithmetic; use input/output function machines to define functions; use function notation *(e.g. f(x)* = 3*x* – 5); manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, by taking out a single term common factor.	**Use output/input inverse function machines; use inverse function notation (*e.g. if f(x)* = 3*x* – 5 *then* $f^{-1}(x) = (x + 5)/3$).** **Expand the product of two linear expressions. (*e.g.* $(x + 1)(x – 2) = x^2 – x – 2$).** **Factorise expressions involving difference of two squares and trinomials.** **e.g** $2x^2+5x-12=(2x-3)(x+4)$. **Use rational expressions with algebraic denominators (*e.g. write* $\frac{1}{x+2}+\frac{x}{x-2}$ *as a single fraction*).**
16. Construct simple linear equations from given situations; solve linear equations; solve simultaneous linear equations in two unknowns, graphically by interpreting the common solution as the point of intersection and algebraically by elimination and by substitution.	**Solve simple linear inequalities in one variable and represent the solution set on the number line (*e.g.* $2x – 3 > 7$). Determine the solution to an inequality or set of inequalities on a graph by shading the appropriate region(s) (*e.g.* $y \geq 3x, y \leq 5$ and $x + y > 4$).** **Solve quadratic equations, by factorisation or by the "quadratic formula".** **Solve a linear equation and a quadratic equation simultaneously.** **Use trial and improvement methods involving calculator and computers to find approximate solutions of equations for which there is not a simple method of solution.** **(*e.g. to solve* $x^3 – x = 80$).**
17. Use formulae arising in mathematics and in other subjects, substitute numbers into a formula, derive a formula and change the subject of the formula including use and construction of a	**Transform more complicated formulae.**

formula on a spreadsheet; use and interpret positive and negative integral indices, including zero; use the index laws in simple instances.	**Use and interpret fractional indices.**
	Solve problems involving direct and inverse variation to determine unknown quantities - restricted to $y \propto x^n$, where n = ±1, ±2, 3.
18. Generate terms of a sequence using term to term and position – term definitions of the sequence; use expressions to describe the *n*th term of a simple sequence.	
19. Demonstrate familiarity with Cartesian coordinates in two dimensions; recognise that equations of the form $y = mx + c$, with m and c specified numerically, represent straight line graphs; construct table of values for linear and quadratic functions; plot and draw graphs of such functions by making use of pencil and paper methods, a spreadsheet and a graphing package; read off values from graphs; understand, interpret and calculate the gradient of a line from the coordinates of two points on it; find the gradient of lines given by equations of the form $y = mx + c$ with both m and c specified numerically; obtain the equation of a straight line in the form $y = mx + c$; know and understand that parallel lines have equal gradients; interpret information presented in a variety of linear and non-linear graphs (*e.g. distance-time and velocity-time graphs; conversion graphs; graphs of height against age*).	**Construct tables of values for cubic functions and reciprocal functions of the type f(x) = a/*x* (*e.g. use pencil and paper, a spreadsheet or a graphing package to generate points and plot graphs of* $y = x^3 - 2x$ *or* $y = 1/x$ *with* $x \neq 0$*);* solve graphically linear, quadratic, cubic and reciprocal functions simultaneously (*e.g. find graphically common solutions for* $y = 2x - 1$ *and* $y = x^3$**).** **Calculate the length of the straight line segment from the coordinates of its end points as an application of Pythagoras' Theorem.**
SHAPE , SPACE and MEASURES 20. Recall and use properties of angles at a point, angles on a straight line, vertically opposite angles; distinguish between acute, obtuse and reflex angles; estimate the size of an angle in degrees. 21. Distinguish between lines and line segments; use parallel lines, alternate angles, corresponding angles and interior angles on same side and between same parallel lines; understand the consequent properties of parallelograms; understand a proof that the angle sum of a triangle is 180°; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices. 22. Use angle properties of equilateral, isosceles and right-angled triangles; explain why the angle	

sum of a quadrilateral is 360°.	
23. Understand and use the essential properties of the square, rectangle, parallelogram, trapezium, rhombus and kite; classify quadrilaterals using their geometric properties.	
24. Calculate and use the sums of the interior and exterior angles of regular and irregular polygons - use a formula, such as [2n – 4] right angles, for the sum of the interior angles of a polygon with n sides.	
25. Understand when shapes are congruent; appreciate the uniqueness of triangles satisfying SSS, SAS, ASA, and RHS; understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles.	**Prove the congruence of triangles in more generalised contexts.** **Understand and use the relationship between lengths, areas and volumes of similar shapes.**
Understand when shapes are similar; understand and use the AAA and common ratio property of sides to prove similarity of triangles.	
Appreciate that all congruent shapes are similar but similar shapes are not necessarily congruent.	
26. Carry out constructions based on measurement; estimate, measure and draw lines and angles; construct parallel lines; construct angles of 60° and 90° using compasses; construct simple geometrical figures from given data; use straight edges and compasses to construct the perpendicular bisector of a line segment, the perpendicular from a point to a line and the bisector of an angle; read and make scale drawings (*e.g to solve right angled triangles*).	
27. Find the perimeter and area of rectangles and triangles by counting unit measures and by formula; find the area of a parallelogram; find the area of a trapezium; find the surface area of simple compound shape; find the circumference and area of a circle; find the length of arc and area of sector as fractions of the circumference and area of a circle; find the surface area of a cube, cuboid, cylinder; find the surface area of a pyramid.	**Find the area of acute and obtuse angled triangles using ½ bc sin A; find the area of segments in a circle; find the surface area of a cone; find the surface area of a sphere.**
28. Find volumes of cuboids by counting unit measures and by formula; find the volume of a prism; find the volume of a cylinder; find the volume of compound shapes involving cubes, cuboids and prisms.	**Find the volume of a pyramid, cone, and sphere. Find the volume of a frustum.**

29. Know and understand the meaning of terms related to the circle: centre, radius, chord, diameter, circumference, tangent, arc, sector, segment.	
30. Use properties of shapes in tessellations; recognise line and rotational symmetry in two dimensions; recognise order of rotational symmetry; recognise properties of triangles, quadrilaterals and circles related to their symmetries; use the symmetry properties of the circle and their converse – equal chords are equidistant from the centre, the perpendicular bisector of a chord passes through the centre, tangents from an external point are equal.	**Prove the symmetry properties of the circle through congruency.**
31. Know and use the following angle properties of the circle to calculate unknown angles – the angle in a semi-circle is a right angle, the angle at the centre is twice the angle at the circumference, angles in the same segment are equal, angles in opposite segments are supplementary, the angle between the radius and the tangent at the point of contact is a right angle. (Reasons justifying the use of these angle facts in simple riders are expected).	**Know and use the alternate segment property.** **Justify, in structured questions, the angle properties of the circle - with the exception of perpendicularity of radius and tangent.**
32. Devise instructions for a computer to produce the desired shapes and paths (*e.g. equilateral triangles and hexagons*); apply the following locus properties in two dimensions in practical situations: • the locus of points which are at a fixed distance from a given point; • the locus of points which are equidistant from two given points.	**Use the following loci in two dimensions:** • **locus of points which are equidistant from a straight line;** • **the locus of points which are equidistant from two intersecting straight lines.** **Use intersecting loci.**
33. Understand, recall and use Pythagoras' Theorem and its converse in 2-D.	**Use Pythagoras' Theorem in 3-D contexts (*e.g. to determine lengths inside a cuboid*).**
34. Understand, recall and use the trigonometrical relationships in right-angled triangles, namely, sine, cosine and tangent, and use them to solve problems in simple practical situations (*e.g .in problems involving angles of elevation and depression*).	**Extend the use of the sine and cosine functions to angles between 90° and 180°; use the sine and cosine formulae to solve any triangle; solve simple trigonometrical problems in 3-D.**
35. Interpret and use three figure bearings measured clockwise from the north; use scale drawing and the trigonometrical ratios to solve problems involving bearings.	
36. Recognise, describe and construct translations, reflections, rotations and enlargements of plane figures.	

Column vectors will be used to describe translations. (In questions requiring candidates to construct transformations on the Cartesian plane, the mirror lines for constructing reflections will be restricted to the axes, $y = \pm x$, $y = \pm c$, $x = \pm c$. The angles of rotation for constructing rotations will be restricted to multiples of 90°. The scale factor for constructing enlargements will be restricted to a positive integer or a fraction.) Recognise that reflections, rotations and translations preserve length and angle, so that any figure is congruent to its image under any of these transformations; appreciate that any two circles and any two squares are mathematically similar, whereas, in general, two rectangles are not; recognise that enlargements preserve angle and not length; understand and use the effect of enlargement of perimeter on 2-D shapes; use and interpret scale drawings.	**Use negative scale factors of enlargement.** **Transform 2-D shapes by a combination of transformations.**
DATA HANDLING 37. Collect, classify and tabulate statistical data (*e.g. gather data from Information and Communication Technology (ICT) sources*); read, interpret and draw simple inferences from tables and statistical diagrams; construct, by pencil and paper and ICT methods, understand and use bar charts, pie charts, simple frequency distributions and histograms with equal intervals; calculate and interpret the range, mean, median and mode for discrete and continuous data; use appropriate statistical functions on a calculator and a spreadsheet.	**Understand and use histograms with unequal intervals; construct and use cumulative frequency curves; estimate the median, the lower and upper quartiles, and the interquartile range from cumulative frequency curves; interpret and construct box plots to illustrate or compare distributions with large data-sets; calculate the mean, median and mode for grouped data and identify the modal class from a grouped frequency distribution.**
38. Calculate the probability of an event; construct simple possibility spaces (*e.g. for the throw of a coin and a die*); work out the combined probability outcomes of two independent events.	**Calculate the probability for combined events, using possibility space diagrams and tree diagrams where appropriate.** **(In tree diagrams outcomes will be written at the end of the branches and probabilities by the sides of the branches.)**